A NOMOGRAM BASED ON THE THEORY OF EXTREME VALUES FOR DETERMINING VALUES FOR VARIOUS RETURN PERIODS

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[Manuscript received March 14, 1955, revised April 15, 1955]

Hydrometeorologists and others interested in the frequency of extreme values of meteorologic or climatic quantities—rainfalls, for example—have found their distribution to be well approximated by a Fisher-Tippett type I distribution of extreme values [1] (sometimes called a Gumbel [2] distribution). The nomogram presented here is based on this distribution and has practical value in the analysis of extreme-value data and the determination of values for various return periods.

The Fisher-Tippett type I distribution is given by

$$P=e^{-e^{-y}} \tag{1}$$

where P is the probability that an extreme value will be less than X, e is the base of the natural logarithms and y is a reduced variate defined following equation (5). $1-P$ is, therefore, the probability that an extreme value will equal or exceed X; and the return period T, is the reciprocal of this probability, i. e.,

$$T=1/(1-P) \tag{2}$$

and

$$y=-\ln \ln(1/P) \tag{3}$$

In this case, the general formula, as given by Chow [3], for frequency analysis can be written,

$$X_T=\overline{X}+S_xK \tag{4}$$

where

$$K=(y-\overline{y}_n)/S_n \tag{5}$$

X=individual item in series of extreme values.

X_T=magnitude of item with return period T.

$\overline{X}$=mean of the extreme values.

S_x=standard deviation of the series of extreme values.

$\overline{y}_n$, S_n=mean and standard deviation of the reduced variate y for the particular sample size n.

y=reduced variate=$(X-u)(S_n/S_x)$.

u=mode of extreme values=$\overline{X}-S_x(\overline{y}_n/S_n)$.

The values of $\overline{y}_n$ and S_n, used in the construction of the graph are shown in table 1. These values are calculated from equation (3), setting $P=m/(n+1)$, where $m=1, 2, 3, \ldots n$.

TABLE 1.—*Computed values of $\bar{y}_n$ and S_n for selected years of record n*

n	10	15	20	30	40	60	100	∞
$\bar{y}_n$	0.4967	0.5128	0.5236	0.5362	0.5436	0.5521	0.5600	0.5772
S_n	.9573	1.0206	1.0628	1.1124	1.1413	1.1747	1.2065	1.2826

The values of the reduced variate, y, for particular values of T, calculated from equations (2) and (3) and used in constructing the graph are shown in table 2, and the values of K calculated from equation (5) and the values in tables 1 and 2, are shown in table 3. Tables 1, 2, and 3 are similar to previously published ones [3, 4] but are extended to lower values of n.

TABLE 2.—*Computed values of y for selected return periods T (years)*

T	2	5	10	25	50	100
y	0.3665	1.4999	2.2504	3.1986	3.9019	4.6002

TABLE 3.—*Computed values of K for selected years of record n, and return periods, T*

T	n						
	10	15	20	30	40	60	100
2	−.1361	−.1433	−.1479	−.1525	−.1552	−.1580	−.1604
5	1.0479	.9672	.9186	.8663	.8379	.8068	.7790
10	1.8319	1.7026	1.6247	1.5410	1.4955	1.4457	1.4010
25	2.8224	2.6316	2.5169	2.3934	2.3263	2.2529	2.1869
50	3.5570	3.3207	3.1786	3.0256	2.9425	2.8516	2.7699
100	4.2865	4.0049	3.8357	3.6534	3.5544	3.4461	3.3486

The relationship given in equation (4) can be put in graph form as shown in figure 1. The nomogram is used in the following way: When the mean and standard deviation of a series of extreme values have been computed, the graph is entered at the desired return period T, thence horizontally to the right to the line of the number of items in the series, thence downward to the line of the standard deviation, thence horizontally to the right to the scale marked D. The value of D read there is added to the mean, $\overline{X}$, to give the value for the required return period T. The graph can be read to within about one percent of the values computed in the usual way.

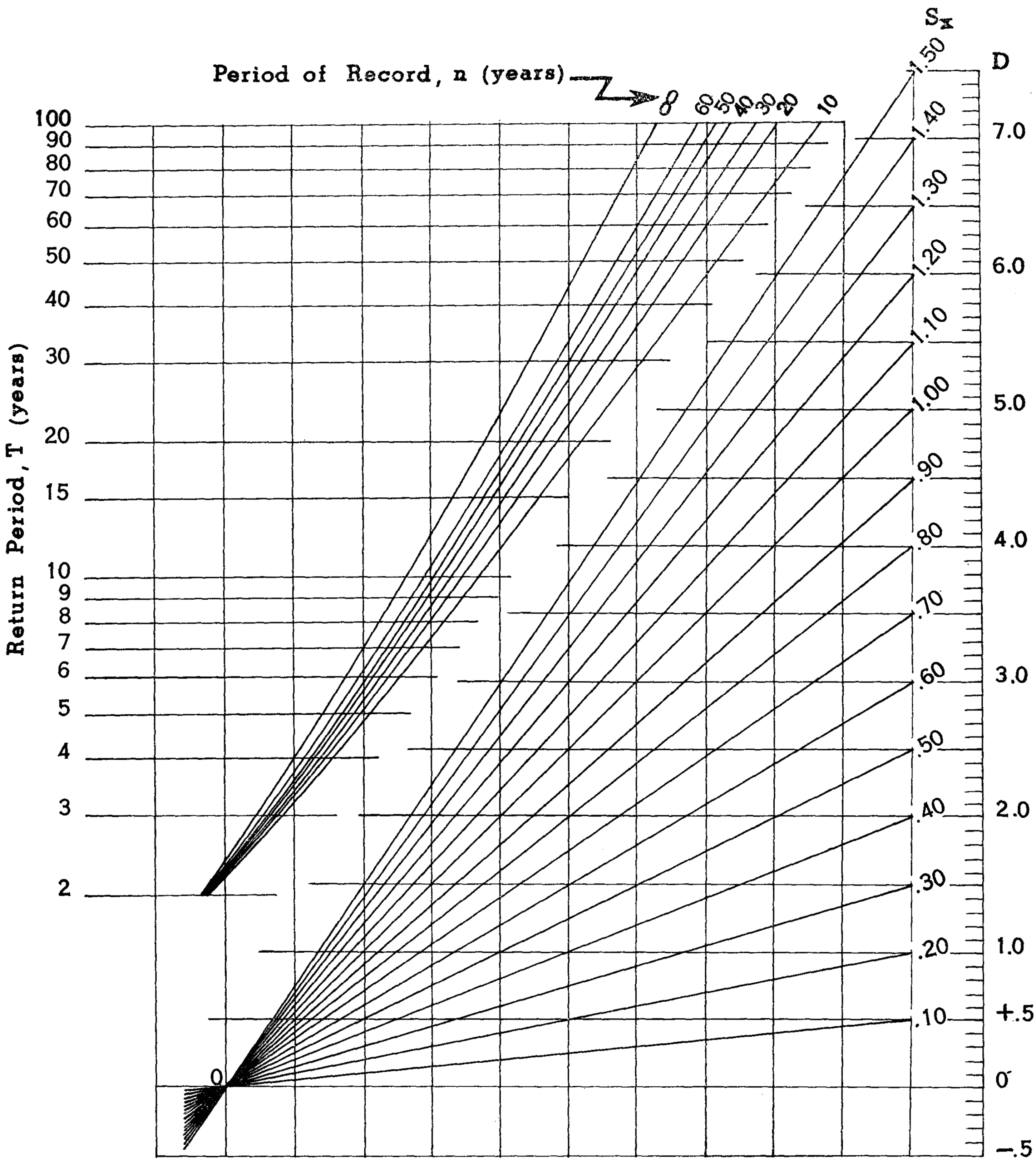

FIGURE 1.—Nomogram to determine D, the amount to be added to the mean of the series of extreme values to get the value for any return period T. The S_x and D scales are relative, i. e., for $S_x=4.00$ use the line labeled .40 and multiply the D value obtained by 10, etc.

Example: Given the mean value of 3.0 and standard deviation of .50 for the series of 40 extremes, required to determine the value that will be equalled or exceeded on the average once in 10 years (i. e., will have a return period of 10 years). Enter left of diagram at $T=10$, move horizontally to line $n=40$, thence downward to line $S_x=.50$, thence horizontally to the right to scale D to read .75, which added to 3.0 gives the answer 3.75.

REFERENCES

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